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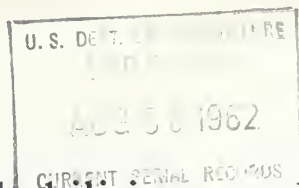
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Yield Probabilities as a Markov Process

By Don Bostwick

The shape and consequences of fortuitous events are subjects of much discussion in economics. Wheat yields on dryland are one member of such a set of events. Much research effort has been devoted to establishment of causal factors in observed yield variability and in attempts to derive probability models without waiting for causal understanding. The discussion that follows is another in the list of attempts of the latter kind. Thanks are due James S. Plaxico of Oklahoma State University, others of the GP-2 Technical Committee, and members of the Farm Economics Division, Economic Research Service staff in Washington, D.C., who read the manuscript and suggested improvements.

MANY of the statistical techniques used to fit yield data to probability models require the assumption of randomness in the distribution of the observational data. This requirement has not been completely satisfactory—an autocorrelation ghost persists in stalking such models, even though hidden in residual error terms.

In the two approaches that follow, the first attempts to invest the ghost with some statistical substance, via a bunchiness test. The second combines a bit of inductive reasoning with the results of the bunchiness tests, toward a hypothesis that autocorrelation of a special sort is not a ghost and as a consequence can be put to work making Markov chains.

Almost any dryland farmer or researcher in the Great Plains who has thought about it believes that wheat yields and/or the weather phenomena underlying them, are *not* an independent and random series over time. On the contrary, these people believe that yields and/or weather phenomena tend to bunch. Bunchiness is the tendency for the phenomena to come in runs of like years, which differ statistically from those expected under random conditions.

R. J. Hildreth of the Texas Agricultural Experiment Station reported tests of the bunchiness hypothesis using annual precipitation in North Dakota over a 40-year period.[1] These tests were somewhat inconclusive.

Greve, Plaxico, and Lagrone report bunchiness tests on production for four enterprises and annual rainfall in northwestern Oklahoma (1).[*] These tests indicated significant bunching of observations in each series of data.

Both of these papers report a nonparametric test developed by Wallis and Moore about 1941. This test requires a long series and a definition of "troughs" and "peaks" of cycles in the series. "In a series of N independent random observations the expected number of completed runs of length d of the signs of first difference ($d+$ signs or $d-$ signs) is:

$$U = \frac{2(d^2 + 3d + 1)(N - d - 2)}{(d + 3)!}\text{ ''}\quad (8, \text{p. }234).$$

The test statistic X_p^2 is computed according to the formula

$$X_p^2 = \frac{(u_1 - U_1)^2}{U_1} + \frac{(u_2 - U_2)^2}{U_2} + \ldots + \frac{(u - U)^2}{U},$$

in which U is calculated as above, and u is the number of runs of length d observed in the data.[2] This is distributed approximately as $6/7$ X^2 for two degrees of freedom if $X_p^2 < 6.3$; for $X_p^2 > 6.3$, it follows the standard X^2 distribution for $2\frac{1}{2}$ degrees of freedom.

A series of 74 county average yields and a series of 63 annual precipitation observations for Hand County, S. Dak. (6, pp. 37–38),[3] were tested for bunchiness. The observation was coded 1 when it was more than 0.5 standard deviation below the mean; 2 when it was within 0.5 standard devia-

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[1] R. J. Hildreth, *Bunchiness of Precipitation Data and Certain Analytical Implications*, paper presented at Methodology Workshop, GP-2 Technical Committee, Lincoln, Nebr., May 5–7, 1959.

[*] Italic numbers in parentheses refer to Literature Cited, page 55.

[2] The variables u and U in the last term of this expression are not subscripted for two reasons: First, there is no necessary limit to the length of runs that may be included in the analysis; second, the last term is open-ended, so the subscript would be of the form, $U_{i \to z}$ rather than the implicit form, $U_{i=z}$ of the previous terms.

[3] Myrick, D. C. An Experiment in Land Use Planning Illustrated by Hand County, South Dakota. Unpublished Ph. D. Thesis, Harvard University, Oct. 1956, pp. 127–128, 369, 378.

tion above or below the mean; and 3 when it was more than 0.5 standard deviation above the mean. A bunch (or run) is defined by a series of identical code numbers. The observed runs were as follows:

Number of runs observed

Runs of years	Yields	Precipitation
1	27	32
2	7	8
3	4	5
4	1	0
5	2	0
6	0	0
7	1	0

The hypothesis under test is that the observed runs are random. The critical value of X_p^2 at the 0.05 level is 5.13; at the 0.10 level, it is 3.95. The calculated X_p^2 for the yield data was 6.17, and for the precipitation data, 3.92. Thus, we can reject the hypothesis of randomness in the Hand County yield data with a 5-percent probability of being wrong. We can reject the hypothesis of randomness in the precipitation data only if we are willing to accept a somewhat greater than 10-percent probability of being wrong. These results agree with the Oklahoma results, at least with respect to bunchiness of yields.

Data on wheat yields on State Lease Land from Judith Basin and Fergus Counties, Mont., gathered by Hjort (2, table II, Appendix A, pp. 71–74), were also tested for bunchiness using the Wallis and Moore procedure. These data were 37 individual yield series with observations in each series for a period of 14 to 23 years, ending with the 1956 crop. I believe that together they are a reasonably representative sample of the geographic area and the span of years. This test is for bunchiness of yields on individual farms over a two-county area; they differ from the preceding tests, which were for bunchiness of county average yields. The 678 observations were distributed in runs as follows:

Runs of	Number observed
1	257
2	80
3	35
4	12
5	5
6	5
7 or more	7

The calculated X_p^2 for these data was 1,747.81. Lumping the last three categories into runs of 5 or more resulted in a calculated X^2 of 199.68. The maximum value of X^2 at the .0005 level is 17.7, so we can be highly confident of bunchiness in the Montana yield data.

The data we have tested for Oklahoma, South Dakota, and Montana support a hypothesis of bunchy yields. This finding compromises the use of traditional probability analyses, which assume randomly distributed sequential observations. What is needed now is a causal hypothesis about the sequential dependence of yields, and a technique for calculating probabilities without violating this dependence hypothesis. They are discussed in the section that follows.

Yield Probabilities as a Markov Process

We have indicated that wheat yields tend to be bunchy or nonrandom in occurrence. A possible explanation for this bunchiness is the use of summer fallow in the wheat rotation. Summer fallow is a device for aiding the growth of a wheat crop from one year to the next. The wheat grown in Montana this year was grown almost entirely on last year's summer fallow, and next year's wheat will come from this year's fallow. So any given wheat crop is functionally related to weather, especially precipitation, over a 2-year period.

A very low yield this year would probably mean low soil moisture storage in the summer fallow. Therefore, next year would need to be very wet for next year's wheat crop to be a bumper one. If the odds favor normal rainfall next year, the wheat crop is likely to be less than normal. Similarly, a very high yield this year should mean good soil moisture in the fallow for next year's crop. The odds should be against a low yield next year; the crop should be normal or better. The hypothesis is that wheat yield on fallow in any year i is a function of precipitation in year i, and in year $i-1$.

This hypothesis happens to be very close to the basic assumption required of a Markov chain. Instead of relating yield to precipitation in sequential years, let us hypothesize that yield in year i is a function of yield in year $i-1$, by way of their common relationship to precipitation in both years. Let us further specify that yield in year i is a random variable with respect to any year in the series other than $i-1$. Year i, of course, is

related to the succeeding year $i+1$, when it will itself be year $i-1$, but it is then a different datum. The hypothesis deals only with sequential pairs of years from what is otherwise a random series.

My favorite example of a Markov chain concerns a frog in a lilypond. The frog is sitting on one of a finite number of lilypads and will presently jump. He may go straight into the air and land back on his starting pad, or he may land on any other pad on the pond. The problem is to assign a probability to the frog's landing on any given pad, starting from any given pad. It is against the rules to land in the water, so there is a probability of 1 (one) that he will land on *some* pad.

The possibilities are shown schematically for a pond containing only three pads.
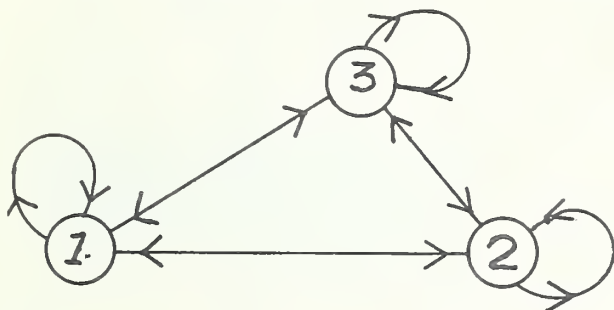
All told, there are nine possibilities, only three of which are pertinent to any one situation. The frog must start somewhere, so the six possibilities that arise from his starting elsewhere are eliminated.

According to our hypothesis, the frog does not jump at random. The probability of his arriving at pad 2, starting from pad 1, differs from the probability of arriving at pad 1 or pad 3. And the probability of arriving at pad 2, starting from pad 1, differs from the probability of arriving at this pad starting from pad 2 or 3. If he has enough jumps, the frog will eventually build a history including jumps from all three pads to all three pads. From these data, all manner of probability statements can be computed. Let us observe a further example of the same process.

A Markov process requires the definition of starting "states" (the pads from which it is possible to jump), and a transition matrix of probabilities attached to jumping from any state to any state in one move. In the frog example, this would be a 3 x 3 matrix.

The example that follows deals with probabilities of dryland wheat yields on a modal Montana farm. Starting "states," in yield per acre, were defined according to the levels of income they would produce. The farm organization—a 1,200-acre dryland wheat farm in north-central Montana, growing 578 acres of wheat each year on summer fallow—was assumed to be constant. Cost data were based on budgets developed by LeRoy Rude (7, Appendix III, pp. 20–22),[4] and a constant price of $1.65 per bushel for wheat was assumed.

Five starting states were defined by the following yields:

1. Less than enough to cover cash costs ($3,815), or $S_1 < 4$ bu./acre.
2. Cover cash costs but less than family living ($6,867), or $4 \leqq S_2 < 7.2$.
3. Cover cash costs and family living but less than interest on investment, or annual debt payments ($17,357), or $7.2 \leqq S_3 < 18.2$.
4. Cover cash costs, family living and annual debt payments, but less than depreciation ($19,932), or $18.2 \leqq S_4 < 20.9$.
5. Cover all costs, leaving a surplus, or $S_5 \geqq 20.9$.

The yield data from Hjort (8) provided 612 paired sequential yield observations with the following distribution:

State in year i	Yield state in year $i+1$					Row sum
	1	2	3	4	5	
1	5	5	14	4	9	37
2	5	11	33	1	4	54
3	12	29	175	30	58	304
4	1	1	32	12	22	68
5	5	4	43	25	72	149
						612

The transition matrix P is computed by setting the sum of each row equal to one and calculating the percentage of the total represented by each component in the row. The transition matrix for these data is:

Starting state	Yield state next year				
	1	2	3	4	5
1	.13514	.13514	.37838	.10811	.24324
2	.09259	.20370	.61111	.01852	.07407
P=3	.03947	.09539	.57566	.09869	.19079
4	.01470	.01470	.47059	.17647	.32353
5	.03356	.02684	.28859	.16779	.48322

Each component is the probability of arriving next year at the state given by the column heading, starting from a state this year indicated by the row heading (p_{ij}=probability of transition from state i to state j). Thus, if yield this year is less than enough to cover cash costs (state 1), the probability of next year's yield falling in the same category (S_1) is .135; and the probability of next year's yield producing a surplus (S_5) is .243, and so on.

This is an ergodic Markov process; it is possible to go from any state to any other in a finite number of steps. The P^2 matrix gives the probability of a given yield state 2 years after a given starting state. It is the square of the transition matrix. The $P^2 \times (P) = P^3$ matrix would give the probabilities 3 years after the start, the $(P^2)^2 = P^4$ matrix, 4 years after the start, and so on.

Start-ing state		Yield state 2 years hence			
	1	2	3	4	5
1	.05546	.09000	.47261	.11435	.26759
2	.05825	.11456	.54140	.08979	.19599
$P^2=$3	.04474	.08625	.50612	.11227	.25062
4	.03537	.06115	.46186	.13373	.30788
5	.03709	.05297	.41364	.14330	.35300

The T matrix is defined as the matrix P^n such that each row is the same probability vector W, and all components of W are positive. These components of W are the probabilities of the process being in any given state at some future year i+n and are independent of the starting state in year i.

There are several methods of computing the T matrix, or W vector. The hard way is to square the P matrix and the successive product matrices until all row vectors are identical, to some limit of precision. This procedure was carried out on an electronic computer to the P^{32} matrix shown below. If a computer is not available, there is a method that shortens the computation involved. This requires multiplying the first row of the P matrix by the P matrix, producing a row that can be called 1′ in a new matrix. Then this row 1′ is multiplied by the P matrix, producing a row 2′, which is again multiplied by the P matrix, producing 3′, and so on. This process is continued until two successive rows of the P′ matrix are identical to some limit of precision. These rows are W vectors.

The third procedure is to compute:

$$\lim_{n \to \infty} P^n = T$$

This was also done on an electronic computer producing the P^n matrix shown below. A comparison of the P^{32} and the P^n matrices gives some indication of the rate of convergence of these yield data.

The stationary vector:

$$W = [.04332 \quad .07655 \quad .47601 \quad .12194 \quad .28215]$$

indicates that the long-run probability of being in yield state 1 (<cover cash costs) is .043+, of being in yield state 5 (a surplus) is .282+, and so on.[5]

Starting State		Yield state 32 years hence			
	1	2	3	4	5
1	.04495	.07939	.49381	.12661	.29373
2	.04424	.07813	.48597	.12460	.28907
$P^{32}=$3	.04419	.07804	.48543	.12446	.28875
4	.04416	.07799	.48513	.12438	.28857
5	.04417	.07802	.48528	.12442	.28866

Starting state		Yield state n→∞			
	1	2	3	4	5
1	.04332	.07655	.47601	.12194	.28215
2	.04332	.07655	.47601	.12194	.28215
$P^n=$3	.04332	.07655	.47601	.12194	.28215
4	.04332	.07655	.47601	.12194	.28215
5	.04332	.07655	.47601	.12194	.28215

If we are now in yield state i, after how many years can we expect to be in state i again? The average expectation of return to any starting state i is the mean recurrence time for that state. This is a vector R, each component of which is the reciprocal of the corresponding component of the fixed (or stationary) vector W (5, p. 413). Thus the mean recurrence vector for the five yield states is:

	Years
1	23.084
2	13.063
R=3	2.125
4	8.201
5	3.544

The next logical step is the computation of the mean first-passage time; this is the mean number of years required to arrive in any given state, starting from any specified state. The mean first-passage matrix is as follows:

Starting state — Mean first-passage time (in years)

	1	2	3	4	5
1	23.0787	14.3486	2.6037	9.0156	4.8929
2	24.1128	13.0616	1.8952	10.0552	5.8865
3	25.7920	15.0420	2.1008	9.1133	5.1555
4	26.6988	16.6093	2.4748	8.2008	4.3565
5	26.2586	16.6581	3.0054	8.1329	3.5441

Note that the values on the major diagonal are approximately those of the mean recurrence vector R above. The differences arise from rounding errors.

A variance and standard deviation estimate can be attached to each of the values in the mean first-passage matrix. The standard deviation matrix is given below.

Starting state — Standard deviation of mean first-passage times

	1	2	3	4	5
1	25.1911	15.1479	2.1143	8.4257	4.6676
2	25.2735	14.8801	1.6683	8.4363	4.6667
3	25.3866	15.1939	1.8974	8.4151	4.6125
4	25.4077	15.2646	2.1359	8.2932	4.4247
5	25.4141	15.2877	2.2764	8.2556	4.1064

The mean first-passage times from these yield data have uniformly large standard deviations. A person using this matrix as a prediction might do well to consider carefully before he stakes a great deal on his prediction.

The reliability of the various probabilities derived from a Markov chain analysis can best be checked by comparison with empirical data. This has not been done with the yield probabilities derived in the foregoing pages. But another way to lend support to a hypothesis, at least for pragmatists, is to describe the many useful applications that would obtain if the hypothesis were valid. The rest of this discussion is an attempt to do just this.

Applications of the Markov Process to Farm Financial Management

A Cash Carryover Decision

How much cash from the year's income should a particular farmer allocate to his cash carryover account? The decision strategy should take account of: (1) the ability to withdraw cash from the crop income this year; (2) the probability of needing cash to cover a short crop next year; (3) the fact that over the long run, no more can be withdrawn from this account than has been put into it; and (4) the undesirability of putting more in than is required over the long run. Let us assume that $3,694 will cover cash operating costs in any given year, and that this is the desired state of the cash carryover account over the long run.

The conditions defined in the paragraph above, and the probabilities from the transition matrix are the raw materials from which a decision formula may be constructed. The formula needs to allow for inputs to and withdrawals from the cash carryover account. These are defined as positive and negative levels of activity in the solution vector. One way of getting a negative level of activity is to subtract one p_{ij} from another, choosing the j's so that the difference is negative when the starting state yield is low and positive when it is high. The result must be multiplied by the modal value assigned to the cash carryover activity, which in this example is $3,694. If the formula does not make use of all components in the ith row and/or manipulates them other than additively, it is necessary to calculate a correction factor that takes account of this omission and manipulation of components.

A generalized formula for the desired strategy is:

$$\text{Cash} = (af(p_{ij}))b$$

in which a is the correction factor, b is the modal value chosen for the activity, and $f(p_{ij})$, states the particular way in which probabilities from the transition matrix are to be manipulated. The value of "a" depends on the $f(p_{ij})$, and is calculated according to the generalized formula:

$$a = \frac{b}{\sum_{i,j=1}^{n} [(f(p_{ij})b)W_j]}$$

in which W_j is the n-valued W vector, and the other terms are as above.

It remains only to state the test formula for the long-run requirement:

$$b = \sum_{i,j=1}^{n} [(af(p_{ij})b)W_j]$$

If the equality in this formula holds, the requirement is met.

For this example, $b=3694$, and $f(p_{ij})=P_{i5}-(p_{i1}+p_{i2})$ was chosen after several trials as one manipulation that met the first two requirements of the desired strategy. Substituting appropriate values in the formula, it was found that $a=6.13734$. This strategy can be stated for any starting state i as:

$$\text{Cash}=6.13734\,(p_{i5}-(p_{i1}+p_{i2}))\,3694,$$

and can be summarized as a decision vector:

$$\text{Cash}=\begin{array}{c}\text{Starting state}\\ \begin{array}{c}1\\2\\3\\4\\5\end{array}\begin{bmatrix}-963.07\\-5038.02\\+1268.00\\+6668.32\\+9585.89\end{bmatrix}\end{array}$$

The strategy is to withdraw from the cash carryover account in the specified amounts if the yield state this year is either 1 or 2, and to add to this account in the specified amount if the yield state is either 3, 4, or 5. The sum of the products of each component in the cash vector and the corresponding component in the W vector is equal to 3693.99, thus fulfilling (within a cent) the long-run requirements for the strategy.

This is not necessarily an optimum strategy. Optimality should be determined only by reference to some overall criteria that include the possibility of interaction between such managerial activities as credit use, stored grain and non-liquid reserve investments, depreciation and machinery replacement, use of insurance, and so on. A cash carryover strategy is not considered here in such a context, but only as an isolated example.

A Fertilizer Decision

The farmer under discussion wants to know whether or not he should apply fertilizer when he seeds his winter wheat crop this fall. Assume that if between seeding and maturity next year rainfall is very low and the rate of fertilizer application is high, the nitrogen will probably cause a reduction in his wheat yield below what could have been expected with no fertilizer. If rainfall is somewhat below normal or near normal, there will be no yield response to fertilizer. If rainfall is above normal, the yield response will be posi-

tive, somewhat in proportion to the increase in rainfall. Lacking data, let us assume that a transition to state 1 will result in a yield reduction of 20 percent, or 3.2 bushels per acre; transition to state 2 will result in a yield reduction of 10 percent, or 6.5 bushels per acre; transition to state 3 leaves yield unaffected; transition to state 4 will result in an increase of 30 percent, or 27.2 bushels per acre; and transition to state 5 will result in a yield increase of 50 percent, or 31.4 bushels per acre. Assume further that the fertilizer can be applied at only one rate, with a fixed cost of $3.50 per acre.

With these data, we can calculate a vector expressing the net gain or loss from fertilizing, if the yield state next year is any one of the five states defined. This vector is:

$$\begin{array}{ccccccc}\text{Net}&1&2&3&4&5\\\text{fertilizer}=[&-2785&-2690&-2023&+3986&+7991]\end{array}$$

These elements must be modified by the starting state (yield this year) and by the probabilities of transition to the five states next year. The sum of the product, element by element, of the net fertilizer vector and a row of the P matrix, will be the properly weighted expectation from fertilizing, given this year's yield. Expressed as a gain vector, it is:

$$\text{Gain}=\begin{array}{c}\text{Starting state}\\ \begin{array}{c}1\\2\\3\\4\\5\end{array}\begin{bmatrix}+827\\-1377\\+387\\+2256\\+3781\end{bmatrix}\end{array}$$

This indicates that the farmer should *not* fertilize this fall if his yield this year was in state 2, that he might as well forgo the effort if this year's yield was in state 3, and that he should fertilize if this year's yield was in state 1, 4, or 5.

For comparative purposes, the fertilizer cost can be increased to $6.50 per acre from the $3.50 used above, and all other data held constant. The gain vector in this situation is:

$$\text{Gain}=\begin{array}{c}\text{Years}\\\text{this year}\\ \begin{array}{c}1\\2\\3\\4\\5\end{array}\begin{bmatrix}-983\\-3125\\-1385\\+457\\+1937\end{bmatrix}\end{array}$$

The increased cost of fertilizer has apparently simplified this farmer's decision process. He will fertilize this fall only if this year's yield state was 4 or 5, that is, only when he has a good to bumper crop.

Allocation of Surpluses

The farmer represented by the data in these examples has a long-run probability of about 0.28 of getting a yield that will provide a surplus of income beyond all costs. He is an adept manager, as indicated by the preceding examples of his decision-making processes. As such, he takes care to use his occasional surpluses to increase his working capital, frittering away very little on excess machinery, family living, and so on. It is assumed that there is no additional land for sale in this farmer's area at a reasonable price, so that he will allocate his surpluses between (1) nonfarm investments, which is a form of financial diversification, (2) stored grain, which provides intermediate-term reserves and concomitantly reduces the income-tax bite in the current year, and (3) cash reserves or consumption expenditures.

This farmer cannot predict the size of a surplus, since state 5 is open-ended (a yield $\geqq$ 20.9 bu./acre), but only that there is a 28 percent probability of *some* surplus. He can use the probabilities from the stationary vector W as an allocating device. The formula might be to put $(w_3 + w_4) = .60$ of the surplus in nonfarm investments, such as stocks and bonds; hold $w_5 = .28$ as stored grain (transferring 28 percent of the potential surplus income to a low-yield state in which the need is greater and the tax less); and hold the remaining $(w_1 + w_2) = .12$ as cash savings or an increment to family consumption expenditures. This strategy applies only when yield this year is in state 5. If he follows this strategy, he might want to develop a strategy for cash carryover that differs from the one suggested earlier. Otherwise, he could end up over the long run with too much cash in reserve at low rates of return.

Conclusions

The last sentence or two above suggest that the three examples used here are all rather arbitrary and would be only partial solutions to a decision process in farm financial management. However, they are intended to illustrate possible uses of

Markov chain analysis, based on yield data, not to suggest real choices. I hope that the examples do suggest the range of possible applications of this probability calculator to problems of decision-making on dryland farms. Perhaps more important is the suggestion prompted by the results of these illustrative examples, that management strategies or decisions based on probabilities from a Markov process might differ decidedly from the choices we customarily scan in the more usual probability analyses.

This discussion has stopped short of a full treatment of Markov processes.[6] I suspect, for instance, that the whole financial management decision process might be fitted into an absorbing chain for the selection of long-range strategies. This would allow absorbing states at the two extremes of bankruptcy and financial success. The problem would be to estimate mean first-passage times to these absorbing states, given some financial state at the start. One could also set up a series of trapping states, representing levels of equity and risk-taking ability, and calculate probabilities of passage time for each state, transition time within each state, and so on.

But these are not as simple as the exercises above. They must be left to further studies. I believe that Markov chain analysis is applicable to yields of wheat on fallow or other dryland cropping systems. It can be a very useful addition to our traditional bag of analytical tools. It might allow us an even closer approximation to the decisions and barnyard analyses of dryland farmers in the Great Plains than is usually the case, and we might really begin to make some significant progress on solutions to their problems.

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[6] See, for instance: R. A. Howard (*3*), who treats of an iterative procedure somewhat analogous to a linear programing solution. He uses the Markov process to attach probabilities to various possible outcomes of selected strategy sets and operation policies.

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